§15. Statistical Theory for Transition and Long-Time Sustainment of Improved Confinement State*

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In this work, the occurrence of stochastic transition is investigated in the presence of triggers by turbulence noise and external events. The probability of observing the transition is calculated under the circumstance that the global controlling parameters change in time. This is another important prediction of statistical theory in addition to the long time average. This clarifies the feature of transient response of the system with stochastic transitions. The interpretation of the experimental threshold database is discussed.

Let's denote the variable of interest by X and the control parameter by C. (X may be chosen as a variable that identify the improved confinement, amplitude of perturbation, etc.) In the case of subcritical excitations like the L-H transition, the deterministic model predicts multiple states of X for given control parameter C. Multiple solutions are denoted by X_A and X_B being separated by X_m. The transition probability has been given as

$$r_{\mathrm{A}\to\mathrm{B}} = \frac{\sqrt{\Lambda_{\mathrm{A}}\Lambda_{\mathrm{m}}}}{2\pi}\exp\left(S(X_{\mathrm{A}})-S(X_{\mathrm{m}})\right)$$

where $S(X)=\int^{X} 4\Lambda(X')g(X')^{-2}X'\,\mathrm{d}X'$ is the nonlinear potential function, and the time rates $\Lambda_{\mathrm{A,\,m,\,B}}$ are given as $\Lambda_{\mathrm{A,\,m,\,B}}=2X\left|\partial\Lambda/\partial X\right|$ at $X=X_{\mathrm{A,\,m,\,B}}$ [1, 2].

The statistical theory gives a new insight for the database of the transition condition. When database for the onset of L-H transition is collected, one measures the value of C when the transition occurs, under the circumstance that the relevant parameter C is gradually increased. This is a kind of transient problem and a new statistical evaluation is necessary, in addition to the long time average. This is obtained in the following. The situation that the first transition is observed is a 'one-through' problem. We consider the situation that the state is in $X=X_A$ at $t=0$ and the controlling parameter C is slowly increasing. The number of ensemble that is in the state $X=X_A$, N_A, satisfies the equation $\frac{\partial}{\partial t}N_A=-r_{A\to B}N_A$. The relative abundance of occurrence of transitions is given by

$$P(C)=\left(\mathrm{d}C/\mathrm{d}t\right)^{-1}r_{A\to B}\exp\left(-\left(\mathrm{d}C/\mathrm{d}t\right)^{-1}\int_{C(0)}^{C} r_{A\to B}\,\mathrm{d}C\right)$$

As C increases in the domain of multiple solutions, the transition from the A-state to the B-state can happen. When C approaches to C_*, which is the phase boundary o the statistical theory [2], transition becomes more frequent. When C far exceeds C_*, the state has already changed to the B-state, and only little A-to-B transition can occur. $P(C)$ has a broad peak around $C=C_*$.
It has been shown that the transition rate changes as $r_{A\to B}(C)\propto\exp\left((C-C_*)/\sigma\right)$ in the vicinity of $C=C_*$ [1, 2]. (The parameter of width σ is given by $\sigma^{-1}=-\int_{X_A}^{X_m}\mathrm{d}X'4X'\,\partial\left\{\Lambda(X')g(X')^{-2}\right\}/\partial C$.) Based on these results, we employ a model $r_{A\to B}(C)\simeq f_\infty\left\{1+\exp\left(-(C-C_*)/\sigma\right)\right\}^{-1}$ where f_∞ denotes the transition rate at $C>>C_*$. We have

$$P(C)=\frac{\sigma^{-1}h\exp\left((C-C_*)/\sigma\right)}{\left\{1+\exp\left((C-C_*)/\sigma\right)\right\}^{1+h}},$$

where $h\equiv\sigma f_\infty\left(\mathrm{d}C/\mathrm{d}t\right)^{-1}$. Figure 1 illustrates the rate of the occurrence of transition [3].

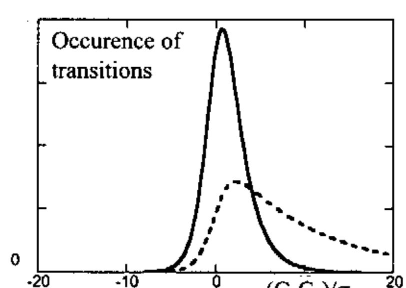

Fig.1 Occurrence of noise-trigger transition (e.g., the L-H transition) when the controlling parameter C is changing in time (a). The case where the change of C is slow ($h\equiv\sigma f_\infty\left(\mathrm{d}C/\mathrm{d}t\right)^{-1}=1$, solid line) and that is fast ($h=0.1$, dashed line) are shown.

References
[1] S.-I. Itoh, K. Itoh, S. Toda: Phys. Rev. Lett. **89** (2002) 215001
[2] S.-I. Itoh, K. Itoh, S. Toda:: Plasma Phys. Contr. Fusion **45** (2003) 823
[3] S-.I. Itoh, K. Itoh, M. Yagi, S. Toda: Plasma Phys. Contr. Fus. **46** (2004) A341